

# **Cost Confidence Interval Estimation: Sensitivity Analysis of Selected Input Distribution Types**

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- Background
- Approach
- Results
- Conclusions

## **Purpose of Cost Confidence Interval Estimation:**

- Determine Confidence Levels for Project Lifecycle Cost Estimates (LCCE)
- Bound the uncertainty around the project cost estimate for Phases A-E/F
- Satisfy KDP-B requirement for a probabilistic analysis of project cost

## **Definition: “S-Curve”**

- A probability distribution for cost that captures the variability/uncertainty in the project cost estimates for each WBS element

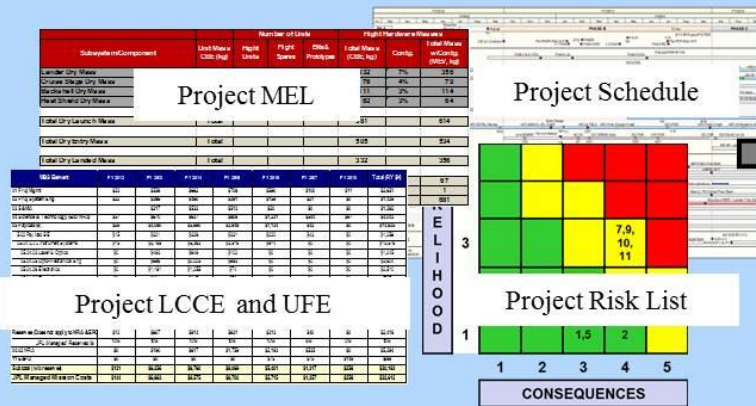
## **Methodology:**

- Monte-Carlo simulation of WBS cost uncertainties to get total cumulative distribution (the “S-curve”)
  - Run simulation and determine confidence levels from the S-curve at the 50th (lower) and 85th (upper) percentiles using a JPL in-house developed tool
  - WBS uncertainties represented by triangular distributions

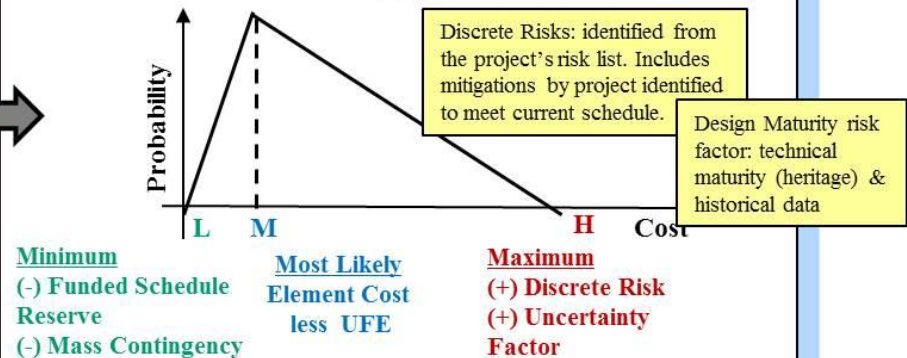
## Question:

*What is the sensitivity of the model results to the assumed input distribution types? If the input distributions were derived from the same data for alternative probability distributions, what, if any differences might be observed?*

## Step 1: Collect Data Sources for Cost Confidence Levels

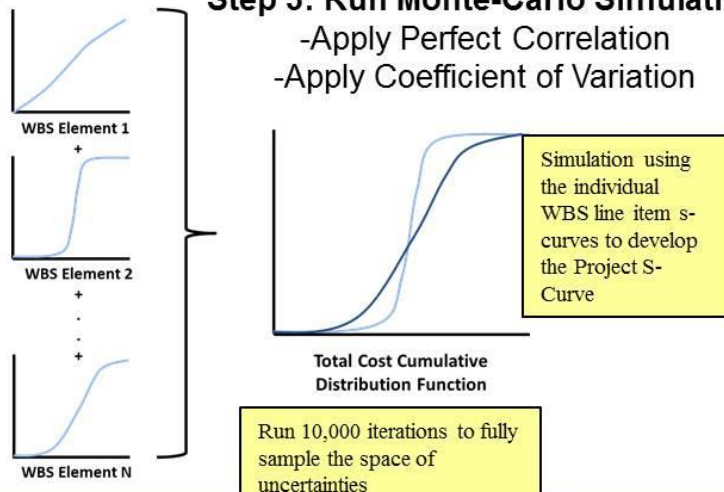


**Step 2: Develop Triangular Distribution** for each WBS Based on Discrete Risks & Uncertainty Factors

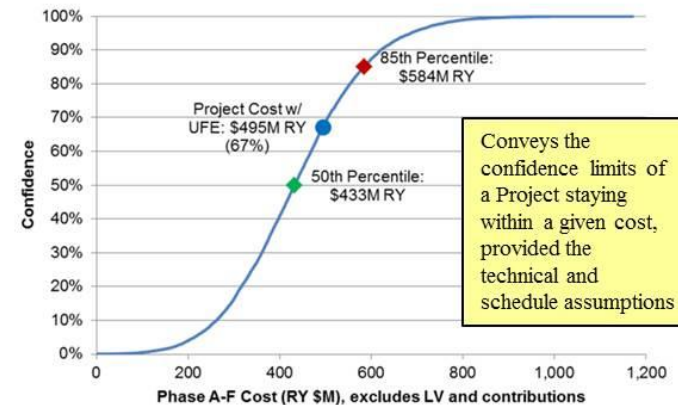


### Step 3: Run Monte-Carlo Simulation

- Apply Perfect Correlation
- Apply Coefficient of Variation



### Step 4: Plot High and Low on S-curve



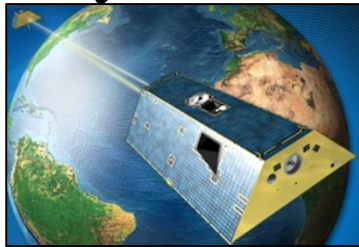
The JPL probabilistic cost approach uses Monte-Carlo simulation:

1. Starts a counter at trial  $n=1$
2. Draws a random cost value from each WBS distribution
3. Computes the total cost for trial  $n$  as the sum of the randomly sampled WBS items.
4. For each simulation trial, the historical variability is added ( $\pm \varepsilon$ )
5. Total costs and statistics for this trial are recorded
6. Repeat for  $n=2, 3, \dots, 10,000$  to fully sample the space of uncertainties
7. Sort from min to max, plot the S-curve and descriptive statistics.

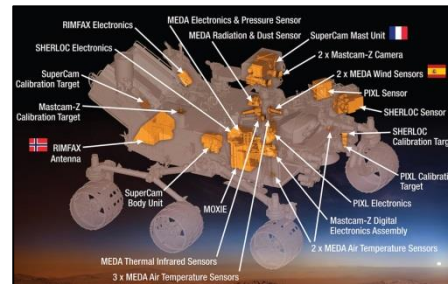
Note: Two cases are run simultaneously—perfect correlation and zero (independent) correlation.

- Experimental approach using simulation experiments
  - Four JPL projects
  - Seven cases compared to Baseline
  - Seven statistical measures for each comparison
  - Full correlation vs. independent correlation

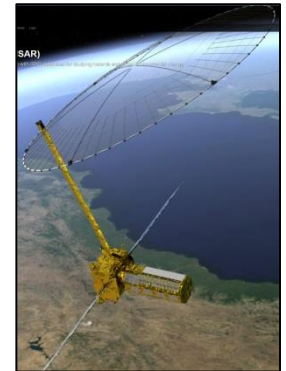
- Gravity Recovery and Climate Experiment Follow-On Project, GFO



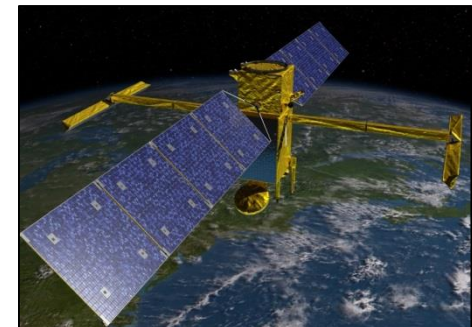
- Mars 2020 mission, M2020



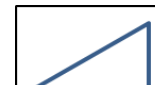
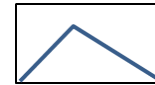
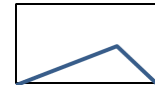
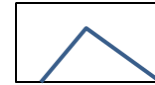
- NASA-ISRO Synthetic Aperture Radar, NISAR



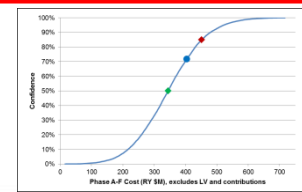
- Surface Water Ocean Topography, SWOT



WBS	Name
01	Project Management
02	Project System Engineering
03	Safety and Mission Assurance
04	Science
05	Science Payload
	...instruments
06	Spacecraft
07	Mission Operations
09	Ground Data System
12	Mission Design
All Phase E/F	All Phase E/F



- Triangular distributions
  - Most likely, min, max
- Simulation model is sum of WBS + adjustments



- If simulation total < Phase A expended, use Phase A amount

Case	Description
Baseline	Triangular distribution inputs for each WBS item with <b>Phase A actuals as minimum.</b>
Case A	Comparison of Baseline to <b>normal distribution with mean = most likely</b> project estimate and standard deviation = standard deviation of the triangular distribution.
Case B	Comparison of Baseline to <b>normal distribution with mean = mean of triangular</b> distribution and standard deviation = standard deviation of the triangular distribution.
Case C	Comparison of Baseline to <b>truncated beta distribution with mean and standard deviation = mean and standard deviation of the triangular distribution.</b> Truncation constrains beta to same range as triangular.
Case D	Comparison of Baseline to <b>non-truncated beta distribution with mean and standard deviation = mean and standard deviation of the triangular distribution.</b> Non-truncated case allows higher and lower range values.
Case E	Comparison of Baseline to <b>uniform distribution</b> with range = range of triangular distribution.
Case F	Baseline using <b>sum of triangular minimums as the lower limit</b> for all simulation estimates.

- 50<sup>th</sup> percentile cost (median),  $\text{Prob}(\text{Total Cost} < X) = 0.50$
- 70<sup>th</sup> percentile cost,  $\text{Prob}(\text{Total Cost} < X) = 0.70$
- 85<sup>th</sup> percentile cost,  $\text{Prob}(\text{Total Cost} < X) = 0.85$
- Average of the Total Cost
- Standard deviation of the Total Cost
- Lower bound, L, of a two-sided 95% confidence interval on the mean,  $\text{Prob}(L \leq \mu \leq U) = 0.95$
- Upper bound, U, of a two-sided 95% confidence interval on the mean,  $\text{Prob}(L \leq \mu \leq U) = 0.95$

Each metric difference from Baseline computed with:

$$\text{Percent difference} = \left( 1 - \frac{\text{Baseline estimate}}{\text{Case estimate}} \right) \cdot 100\%$$

- 4 projects x 2 correlation modes x 7 cases x 7 metrics = 392 comparison values.
- Results compiled in two ways
  - Tabular form displaying actual percent differences (both positive and negative)  $> 1\%$ .
  - Radar charts displaying absolute differences of all 7 metrics simultaneously for each case. Full correlation case shown here (independent case was similar)

# Results by Case Part 1/3



Case A: Baseline vs. Normal distribution with mean=most likely; std dev = triangular

Full Correlation	95% CI on mean						
	50% -tile	70% -tile	85% -tile	Mean	Std Dev	Lower	Upper
GFO	-3%	-2%	-2%	-2%	-	-2%	-2%
NISAR	-11%	-9%	-7%	-11%	-	-10%	-11%
M2020	-10%	-11%	-11%	-11%	-9%	-11%	-11%
SWOT	-11%	-11%	-11%	-11%	-9%	-11%	-11%

Independent Correlation	95% CI on mean						
	50% -tile	70% -tile	85% -tile	Mean	Std Dev	Lower	Upper
-1%	-2%	-1%	-1%	-2%	-1%	-5%	
-11%	-10%	-6%	-10%	-1%	-10%	-10%	
-11%	-11%	-11%	-11%	-12%	-11%	-11%	
-12%	-11%	-11%	-11%	-10%	-11%	-11%	

Case B: Baseline vs. normal distribution with mean=triang. and std dev = triang.

Full Correlation	95% CI on Mean						
	50% -tile	70% -tile	85% -tile	Mean	Std Dev	Lower	Upper
GFO	-	-	-	-	-	-	-
NISAR	-	-	-	-	-	-	-
M2020	-	-	-	-	-	-	-
SWOT	-	-	-	-	-	-	-

Independent Correlation	95% CI on mean						
	50% -tile	70% -tile	85% -tile	Mean	Std Dev	Lower	Upper
-	-	-	-	-	-	-	-
-	-	1%	-	-1%	-	-	-
-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-

*Explanation in a moment...*

Case C: Baseline vs. beta distribution truncated to [a,b]

Full Correlation			95% CI on Mean				
	50%-tile	70%-tile	85%-tile	Mean	Std Dev	Lower	Upper
GFO	-	-	-	-	3%	-	-
NISAR	-	-	1%	-	3%	-	-
M2020	-	-	-	-	2%	-	-
SWOT	-	-	-	-	1%	-	-

Independent Correlation			95% CI on mean				
	50%-tile	70%-tile	85%-tile	Mean	Std Dev	Lower	Upper
	-	-	-	-	-	-	-
	-	-	-	-	-	-	-
	-	-	-	-	2%	-	-
	-	-	-	-	2%	-	-

Case D: Baseline vs. beta distribution non-truncated.

Full Correlation			95% CI on Mean				
	50%-tile	70%-tile	85%-tile	Mean	Std Dev	Lower	Upper
GFO	-	-	1%	-	3%	-	-
NISAR	-	-	-	-	3%	-	-
M2020	-	-	-	-	2%	-	-
SWOT	-1%	-	-	-	3%	-	-

Independent Correlation			95% CI on mean				
	50%-tile	70%-tile	85%-tile	Mean	Std Dev	Lower	Upper
	-	-	-	-	-	-	-
	-	-	-	-	-	-	-
	-	-	-	-	2%	-	-
	-	-	-	-	1%	-	-

*Explanation in a moment...*

## Results by Case Part 3/3



Case E: Baseline vs. uniform distribution [a,b]

Full Correlation				95% CI on Mean			
	50% -tile	70% -tile	85% -tile	Mean	Std Dev	Lower	Upper
GFO	-	-	-	1%	6%	1%	1%
NISAR	5%	6%	6%	5%	6%	5%	5%
M2020	5%	5%	6%	5%	10%	5%	5%
SWOT	4%	5%	6%	4%	9%	4%	4%

Independent Correlation				95% CI on mean			
	50% -tile	70% -tile	85% -tile	Mean	Std Dev	Lower	Upper
-	-	-	-	-	-	-	-
4%	5%	5%	5%	5%	-	5%	5%
5%	5%	5%	5%	5%	6%	5%	5%
4%	5%	5%	5%	5%	8%	5%	5%

Case F: Baseline using actuals (Phase A) as minimum vs. sum of triangular minimums

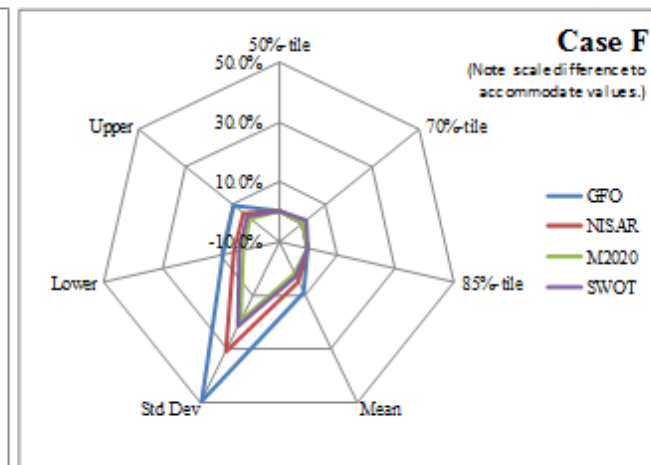
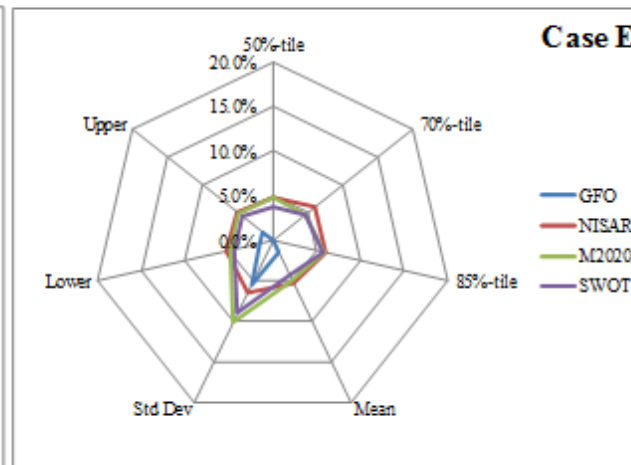
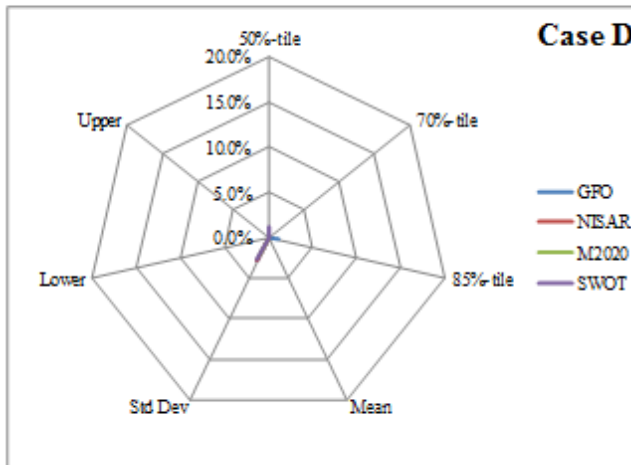
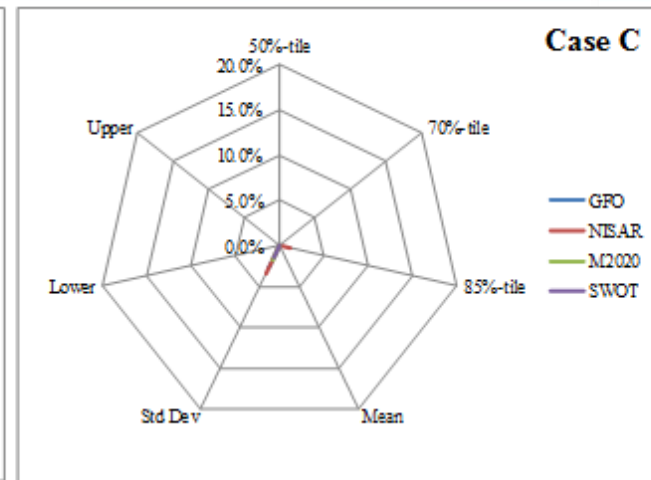
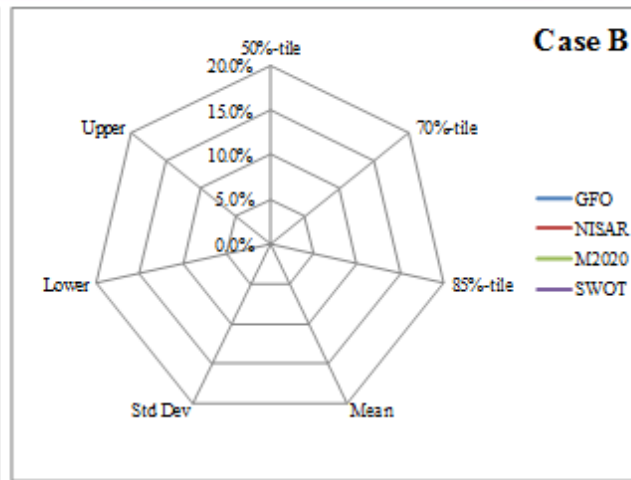
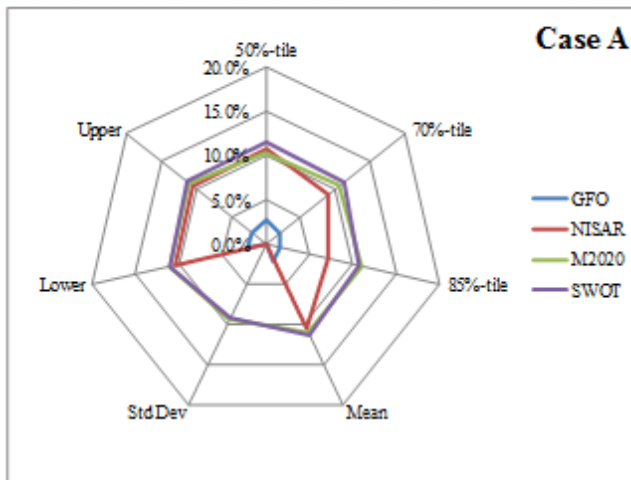
Full Correlation				95% CI on Mean			
	50% -tile	70% -tile	85% -tile	Mean	Std Dev	Lower	Upper
GFO	-	-	-	9%	-49%	9%	9%
NISAR	-	-	-	5%	-31%	5%	5%
M2020	-	-	-	2%	-20%	2%	2%
SWOT	-	-2%	-	3%	-21%	4%	3%

Independent Correlation				95% CI on mean			
	50% -tile	70% -tile	85% -tile	Mean	Std Dev	Lower	Upper
-	-	-	-	4%	-22%	4%	4%
-	-	-	-	5%	-32%	5%	5%
-	-	-	-	2%	-20%	2%	2%
-	-	-	-	4%	-22%	4%	4%

*Explanation in a moment...*

Case	Description
Case A	Normal using mean = most likely ignores skewness of triangular (and cost) which centralizes the estimate artificially. Impact: estimate and standard deviation reduced.
Case B	Normal using mean of triangular distribution is an excellent fit but due mainly to effect of Central Limit Theorem cancellation of errors. Impact: Good fit but skewness of inputs lost in translation.
Case C	Truncated Beta fits well with slight increase in standard deviation (3%). Impact: Preserves shape of triangular closely with minor exception in standard deviation.
Case D	Non-truncated Beta makes no significant difference from truncated case. Impact: does allow lower and higher values than the truncated case without affecting the percentiles or mean.
Case E	Uniform distribution incurs differences up to 10% due to loss of mode (most likely value).
Case F	Use of sum of triangular minimums as the lower limit for all simulation estimates generates significant differences up to almost 50%.

# Results Visualization



Distance from center indicates magnitude of absolute value difference by metric

- Only four projects were evaluated
- The Baseline for each project was an estimate—the comparisons were not validated against actuals from completed missions.
- Other comparison metrics (kurtosis, mean square error, etc.)
- Other distribution types: Gamma, Tri-gen, lognormal

- The triangular distribution was the most tractable distribution for the purposes of spreading the uncertainties from a project point estimate into a probability distribution.
- No significant advantage could be found by switching to the normal, beta, or uniform distribution types. Switching to the uniform distribution actually results in a loss of information since there is no mode.
- Using the sum of minimum costs as the lower bound on total cost creates a bias in the mean since it cuts off portions of the lower tail which artificially reduces the standard deviation (~50% in some cases). Further study would be needed using actual costs to quantify the effect with greater precision.

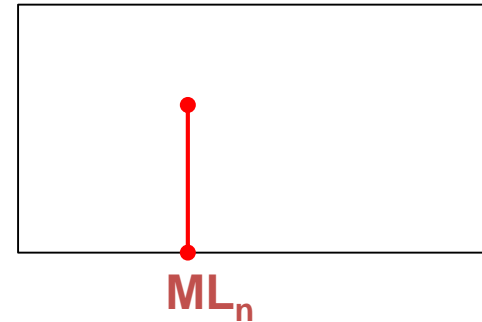
# Backup

# Computing the “Spread” for Triangular Distributions



Step 1: Most likely value for

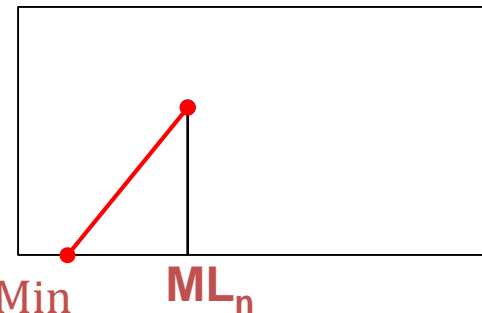
WBS item n = Project cost estimate =  $ML_n$



Step 2: Minimum: deduct mass contingency percent,  $m_n$ ,

and schedule reserve,  $s_n$  = funded schedule

reserve (months) x monthly burn rate (\$/month):

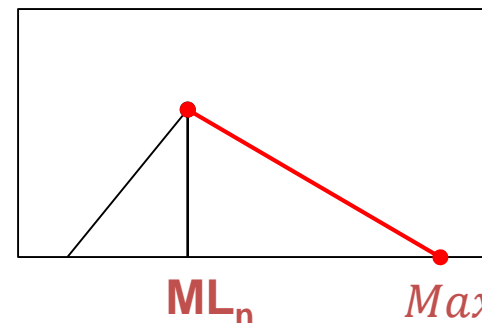


$$\frac{ML_n}{(1+m_n)} - s_n = \text{Min}$$

Step 3: Maximum: Add total risk cost impact from risk

list assigned to this WBS item,  $r_n$ , and escalate result by

the design maturity risk factor,  $d_n$ :



$$= (ML_n + r_n) \cdot (1 + d_n)$$